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THE CHOICE OF THE SYSTEM OF GEOMETRIC PARAMETERS OF AN OPTIMIZED WING†

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A direct optimization method is used to determine the form of the wing which enables the aerodynamic performance to be improved for a given lift in the supersonic flow of ideal gas. The flow around the wing and its characteristics are calculated within the framework of a model based on Euler's equations. On the basis of a local analysis of the load distribution on the wing, a method is proposed for choosing the system of geometric parameters which ensures rapid convergence to the optimum. It is shown that one of the parameters of the system (the angle of rotation of the wing panel relative to the central chord) has a very slight influence on the aerodynamic characteristics of the wing. © 1999 Elsevier Science Ltd. All rights reserved.

When designing wings with optimum characteristics at supersonic flight speeds, the answers to many questions may be obtained by using fairly simple flow models, including models based on the linearized equations of motion [1, 2]. In particular, it has been shown that a conical deformation of a wing of triangular plan form gives a marked increase in aerodynamic performance for a given lift, compared with a triaxial deformation, which gives a much smaller increase in aerodynamic performance. More precise data may be obtained using the Euler flow model and, in the high supersonic velocity range, thin shock layer theory [3, 4].

Direct optimization methods, combining methods of computational aerodynamics with numerical optimization techniques, seem to be the most promising. In such methods one has to scan a large number of versions, so that the demands on the available computer resources are considerable, particularly when the number of independent parameters defining the aircraft geometry is large. Nevertheless, direct design methods have the important advantage that they enable the design process to be monitored. In some cases, basing oneself on the principle of gradually complicating the wing form and monitoring the resultant changes in aerodynamic performance, one can determine the class of simplest deformations [3]. The efficiency of direct optimization methods may be improved significantly by a proper choice of the system of geometrical parameters.

1. The problem is to determine, at a given Mach number, the lift coefficient C_y the leading-edge sweep angle χ which gives maximum aerodynamic performance K, and the corresponding wing form. We will consider a class of infinitesimally thin triangular wings whose surfaces are formed by plane elements joined together along rays emanating from the apex. The number of plane elements was varied from two to 32. The aerodynamic performance of a wing of N elements will be denoted by K_N. Figure 1 shows a wing with eight plane elements.

The wing surface is deformed in such a way as to preserve the lengths $|OA_1| = |A_1B_1| = |B_1C_1| = |C_1D_1|$, where A_1, B_1, C_1 and D_1 are the projections of the corner points A, B, C and D onto the base plane (which coincides with a flat wing). The wing platform is identical with the form of the initial flat wing. The surface are around which the gas flows was not required to remain unchanged. Thus, the wing geometry is uniquely defined by the vertical displacements h_1, h_2, h_3 and h_4 of the points A, B, C and D relative to the base plane. It s assumed that a positive increment to the parameters h_1, h_2, h_3 and h_4 corresponds to leeward displacement of the points. When computing the coefficients of the aerodynamic forces, the characteristic area was defined to be that of a flat wing.

In the case in question, the aerodynamic performance and lift coefficient of the wing are functions of the geometric parameters and the angle of attack. Thus, our problem is to maximize a function of several variables, subject to an additional condition represented by the equality

$$K_8(h_1, h_2, h_3, h_4, \alpha) = \max, \; Cy(h_1, h_2, h_3, h_4, \alpha) = \text{const}$$

where α is the angle of attack.

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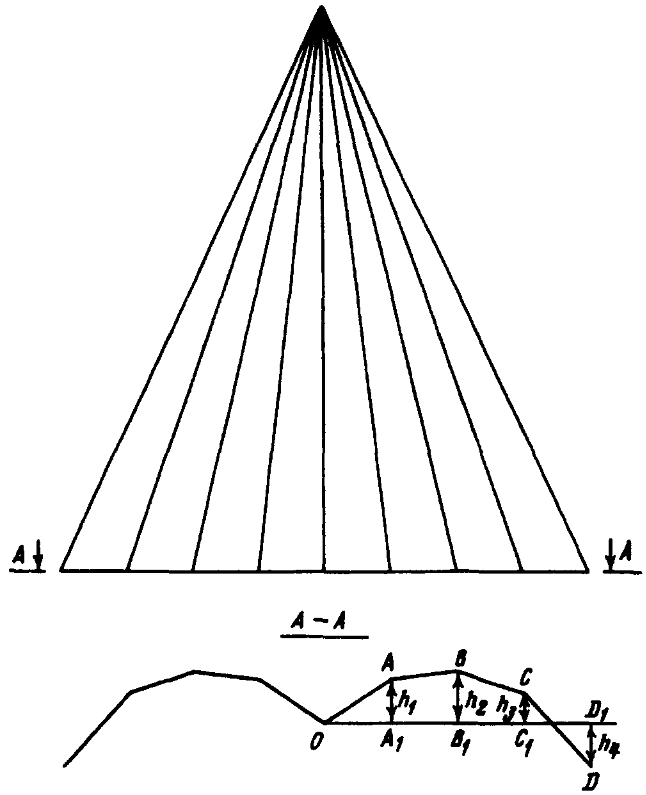

Fig. 1.

Since this study is limited to the range of moderate angles of attack, the derivative of the lift coefficient with respect to the angle of attack is positive, and for each assignment of values to the geometric parameter there is a unique value of α that guarantees conservation of the lift. One can thus use methods approved for problems of unconditional maximization (minimization) of functions of several variables in an unbounded domain.

2. Not one of the numerical optimization techniques used to solve aerodynamic design problems enables one completely to avoid the difficulties involved in investigating objective functions of complex topography, such as "ravines" and local minima and maxima. Complex topography is often a consequence of failure to make allowance for some regularity in a relationship among the variables. Successful solution of the optimization problem will therefore depend to a significant degree on the correct selection of the parameter system.

In this paper the maximum aerodynamic performance and the corresponding values of the geometrical parameters have been determined by the method of coordinate-wise descent. This method consists of the successive determination of the extremal values of the independent variables. The process is repeated until it converges to certain optimum values, where further variation of the parameters produces no further increase in aerodynamic performance. At the same time, the angle of attack which guarantees conservation of the lift coefficient was found by interpolation, based on data obtained by calculating the flow around the wing at angles of attack for which the computed values of the lift coefficient approximated the given values.

The method of coordinate-wise descent is easily computerized. However, if the independent variables are chosen arbitrarily, the method does not always converge rapidly to an optimum. For the problem considered here, we were able to determine a system of geometric parameters, variation of which gave a marked acceleration in the convergence of the process.

The parameter system was chosen on the assumption that the following quantities are small: the angle of attack ($\alpha \to 0$), the wing deformation ($h_i/l \to 0$, $h_i \operatorname{tg} \chi/l \to 0$, $i = 1, 2, \ldots, n$, where l in the wing length and $n = N/2$ the number of geometric parameters); we also assumed the validity of the "strip" theory, according to which a three-dimensional body is divided into strips by parallel planes in the free-stream direction and it is assumed that there is no interaction between the strips. The load on each strip was determined by Ackeret's formula for linearized supersonic flow around a thin profile.

In the case of a wing formed by eight elements, the following equations describe the relation among the values of the local angles of attack of the individual elements

$$\alpha_k = \alpha_1 + ((k-1)h_k - kh_{k-1})/l, \; k = 2, 3, 4$$

The first element of the wing is that closest to the plane of symmetry and the last is the element farthest from the plane of symmetry. The wing length l, which occurs in the equation, is a normalizing coefficient for the parameters h_1, h_2, h_3 and h_4, and may be equated to unity without loss of generality.

The condition of lift conservation readily yields the following relations for the local angles of attack as functions of the geometric parameters and the angle of attack α_f of a flat wing

$$\alpha_1 = \alpha_f + (2h_1 + 2h_2 + 2h_3 - 3h_4)/(4l)$$
$$\alpha_2 = \alpha_f + (-6h_1 + 6h_2 + 2h_3 - 3h_4)/(4l)$$
$$\alpha_3 = \alpha_f + (2h_1 - 10h_2 + 10h_3 - 3h_4)/(4l)$$
$$\alpha_4 = \alpha_f + (2h_1 + 2h_2 - 14h_3 + 9h_4)/(4l)$$

Under the assumptions made here, the load on each element of the wing is directly proportional to the local angle of attack. Therefore, the direction at which the load on an element increases most rapidly is the direction of the gradient of the function of local angle of attack. Conversely: if one moves in a direction perpendicular to the gradient, the load on the element remains unchanged. In the case under consideration we have the following expressions, accurate to three significant figures, for the gradients (per unit length) in the space defined by the geometric parameters (h_1, h_2, h_3, h_4)

$$\mathbf{g}_1 = (0.436;\ 0.436;\ 0.436;\ -0.655)$$
$$\mathbf{g}_2 = (-0.651;\ 0.651;\ 0.217;\ -0.325)$$
$$\mathbf{g}_3 = (0.137;\ -0.685;\ 0.685;\ -0.206)$$
$$\mathbf{g}_4 = (0.118;\ 0.118;\ -0.829;\ 0.533)$$

These four vectors are not linearly independent—as a consequence of the condition of conservation of lift—and a unique vector (apart from sign) $\mathbf{g}_0 = (0.183;\ 0.365;\ 0.548;\ 0.730)$ orthogonal to each of them exists. This vector corresponds to the rotation of the wing panels relative to the central chord. If one moves in the direction defined by the vector $\mathbf{g}_0$, the load on the wing varies only slightly. Thus, we have defined a geometric parameter representing the stability of the aerodynamic characteristics of the wing.

In a small neighbourhood of the optimum point, the increment to a sufficiently smooth function is usually expressed as a quadratic form

$$K_8(r_1, r_2, r_3, r_4) - K_8(r_{10}, r_{20}, r_{30}, r_{40}) = \sum_{i=1}^{4} A_i (r_i - r_{i0})^2$$

where r_i are the geometric parameters (henceforth we shall call them the basic parameters) and r_{i0} are their optimum values. In the case of a local or global maximum, the quadratic form is negative definite, that is, the coefficients A_i are not positive. The problem is to determine a system of basic geometric parameters, after which the search for the optimum will be considerably simplified: we need only vary each parameter separately.

One such parameter is the stability of the aerodynamic characteristics. It exerts the least influence on the characteristics of the wing and will be taken as the last (in this case, fourth) parameter of the system. We propose to determine the other parameters from the following conditions. We first establish the optimum load on the wing element nearest the leading edge, then on the next element (on the assumption that the load on the already optimized element remains fixed) and so on. Thus, the vector of the third parameter of the system must be orthogonal to the vectors $\mathbf{g}_0$, $\mathbf{g}_3$ and $\mathbf{g}_4$. The vector of the second parameter is orthogonal to $\mathbf{g}_0$, $\mathbf{g}_4$ and the vector of the third parameter. Finally, the vector of the first parameter must be orthogonal to the vectors of the second, third and fourth parameters. Hence we obtain the following relations among the basic geometric parameters and the parameters h_i

$$r_1 = (0.118h_1 + 0.118h_2 - 0.829h_3 + 0.533h_4)/l$$
$$r_2 = (0.336h_1 - 0.892h_2 + 0.103h_3 + 0.284h_4)/l$$
$$r_3 = (0.916h_1 + 0.239h_2 - 0.040h_3 + 0.319h_4)/l$$
$$r_4 = (0.183h_1 + 0.365h_2 + 0.548h_3 + 0.730h_4)/l$$

The coefficients in the equations have been chosen in such a way that a unit vector in the space of the basic parameters is associated with a vector of length l in the space of the initially chosen parameters.

3. The flow of an ideal gas around the wing is conical and was calculated using the time-convergence method with respect to the longitudinal coordinate. The steady equations of motion were written in conservative form, so that it was possible to obtain correct information about the density jumps and other flow discontinuities without specially monitoring their spatial positions. Euler's equations were integrated using McCormack's explicit finite-difference scheme, employing the multi-zone approach when constructing the computation mesh [5]. The flow region under investigation was divided into zones, similar in shape to quadrilaterals, located above and below the wing. The zone dimensions were chosen in such a way that the disturbances of the flow due to the wing did not go beyond the zones. At the outer boundaries of the zones, therefore, the given data were the gas-dynamic variables for undisturbed flow. At mesh points on the wing surface it was assumed as a boundary condition that the surface was impermeable. The well-known principles of reflection were assumed to hold in the plane of symmetry.

For the optimization computations we used a mesh which, in each of two zones, had 41 mesh-points in the direction of the normal to the wing plane and 77 mesh-points in the direction of the normal to the plane of symmetry. In addition, 49 mesh-points were placed on each of the lower and upper surfaces of the wing panel. To confirm the results, the mesh was refined by factors of two and four (in each direction). In the latter case the total number of mesh-points of the computation mesh was approximately 100,000 in each cross-section.

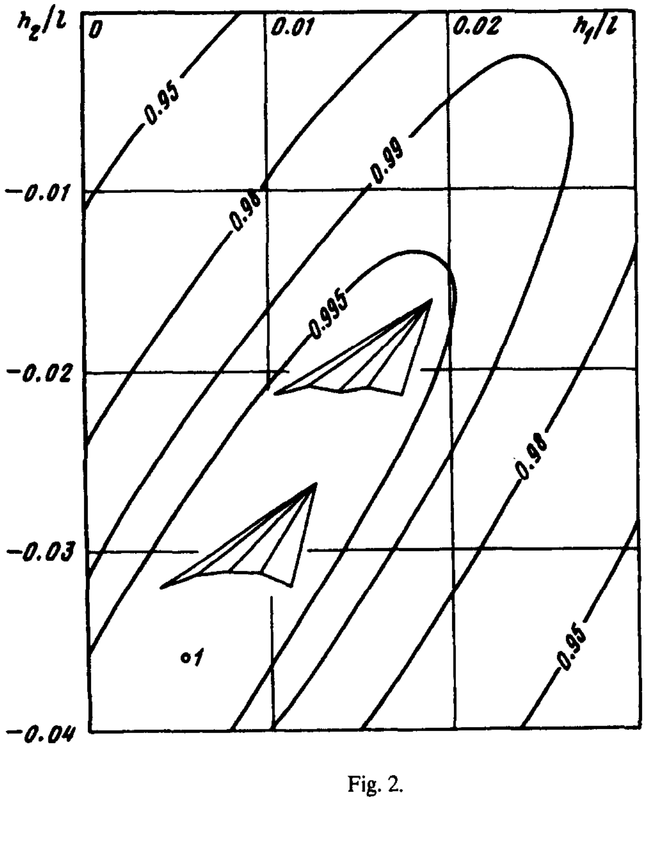

Fig. 2.

4. The optimization studies were carried out for a wing with a leading-edge sweep angle $\chi = 60°$ at $M = 2$ and for a wing with angle $\chi = 75°$ at $M = 4$. The lift coefficient was $C_y = 0.1$. In both cases the results were qualitatively identical.

The aerodynamic performance (AP) of a wing consisting of four plane elements depends on two parameters, and the relief of the surface thus defined may be represented by level curves (Fig. 2 shows the case of a wing with $\chi = 75°$ at $M = 4$ and $C_y = 0.1$. The elliptic shape of the level curves indicates a trough-shaped relief and confirms the validity of the previous assumption, according to which the change in the AP is represented by a quadratic form. Because of the strong elongation of the curves and the steep inclination relative to the coordinate axes, convergence of the coordinate-wise descent in h_1 and h_2 is slow. However, it is known from previous analysis that the level curves are elongated in the direction defined by the geometric stability parameter. One can therefore make a change of variables and move in the directions of the basic geometric parameters—$r_1 = (-0.894h_1 + 0.447h_2)/l$ and $r_2 = (0.447h_1 + 0.894h_2)/l$, which are practically parallel to the axes of the ellipses. This guarantees rapid and precise determination of the optimum. We note that when $M = 2$ the level curves are even more elongated than when $M = 4$. In that case, coordinate-wise descent in h_1 and h_2 does not guarantee convergence to the optimum point and a change of variables becomes unavoidable.

From a practical point of view, the geometric stability parameter manifests itself in the existence of wings which differ considerably in shape but possess the same aerodynamic characteristics. Figure 2 illustrates wings of the same AP—0.5% less than the maximum AP for this class of wings; however, the parameter h_1 in one wing is positive, while that in the other is negative.

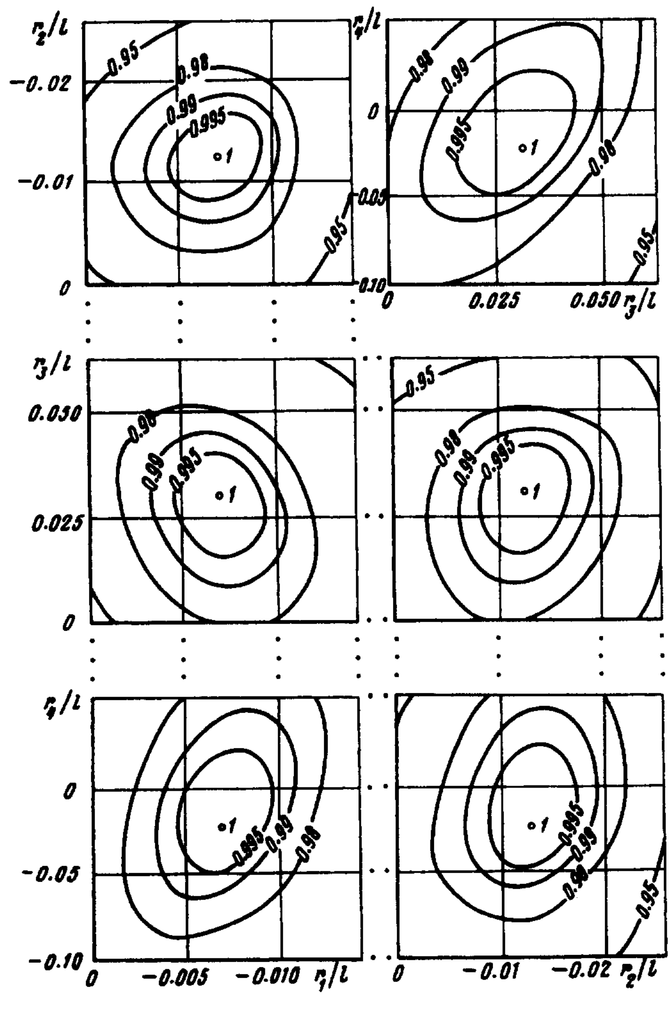

Fig. 3.

The method proposed earlier for constructing a system of basic geometric parameters has also proved successful for wings with a larger number of generating elements. Satisfactory convergence to the optimum has been achieved in two to four cycles of coordinate-wise descent.

Figure 3 shows level curves of the AP for a wing consisting of eight flat elements ($\chi = 75°$, $M = 4$ and $C_y = 0.1$), in the six planes through the optimum point defined by the different pairs of basic geometric parameters r_i. The level curves are elliptic in shape and slightly inclined to the coordinate axes. This indicates rapid convergence of the descent with respect to the basic geometric parameters.

The successful choice of the system of geometric parameters enables one easily to derive an analytic expression for the approximate computation of the AP of a wing of arbitrary form. Processing the results of the numerical computation for $M = 4$ gives the following relation

$$1 - K_8/\max K_8 =$$
$$= 705(r_1 + 0,00712)^2 + 223(r_2 + 0,0126)^2 + 18(r_3 - 0,0305)^2 + 4(r_4 + 0,0242)^2$$

It is clear that the parameter whose variation exerts the most influence on the AP is r_1, corresponding to the load on the wing element adjacent to the leading edge. The AP changes least when the stability parameter r_4 is varied. In the space of the initially chosen geometric parameters, the AP depends only

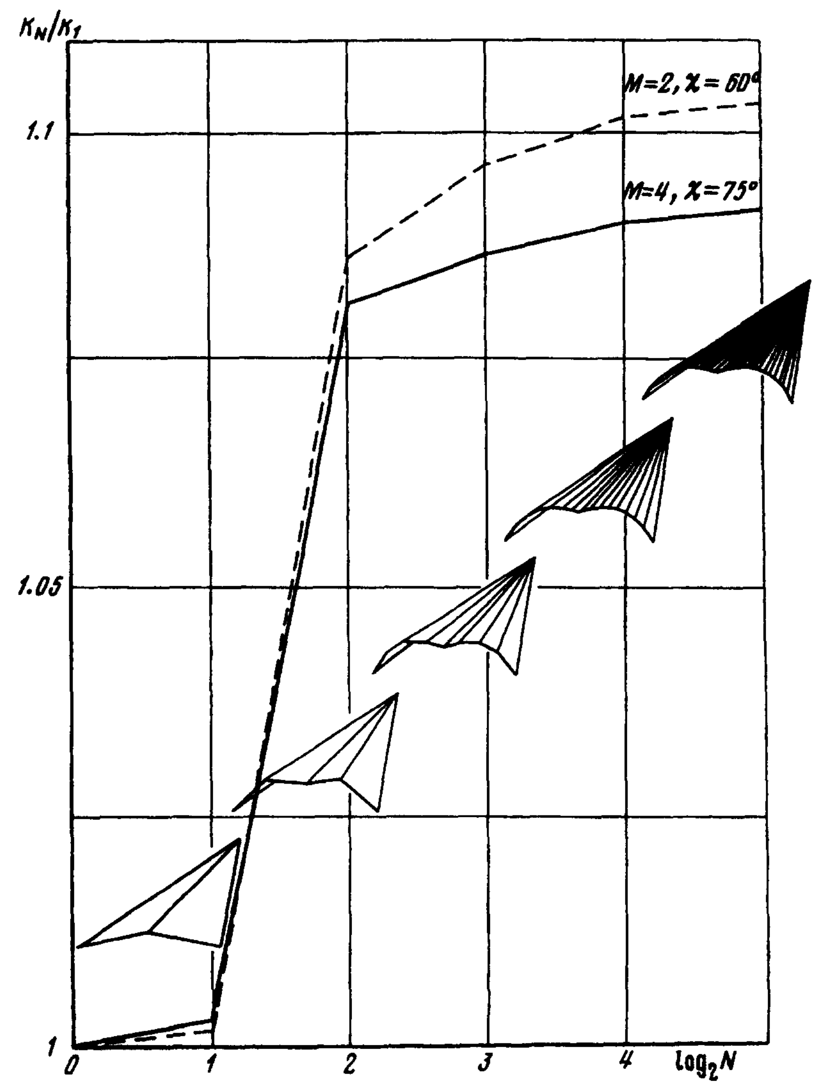

Fig. 4.

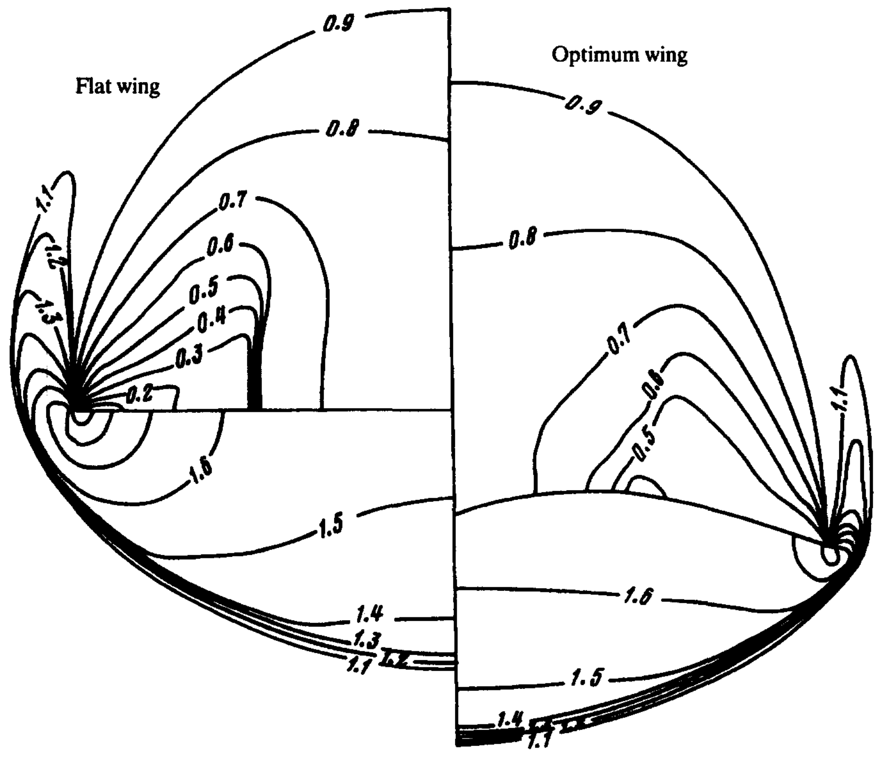

Fig. 5.

slightly on the parameter h_1, which defines the form of the wing near the plane of symmetry, and strongly on h_3, which affects the form in the neighbourhood of the leading edge.

The above relation guarantees satisfactory accuracy when determining the AP over a wide range of variation of the geometric parameters. Thus, for a flat wing one has a decrease in 9% of the AP compared with the optimum wing. Numerical computation for this case gives a value of 8.1%.

The principle employed in this paper of gradually complicating the wing form, enabled us to analyse the nature of the variation of the AP as a function of the number of plane elements at $C_y = 0.1$ (Fig. 4). It is obvious that by simply making the wing V-shaped one obtains practically no increase in the AP. This is because in that case only the stability parameter is varied. The largest gain in the AP occurs on changing from wings with two plane elements to wings with four elements. Further complication of the wing form, that is, a further increase in the number of generating elements, yields a significantly lower gain. This implies that, in practice, it is best to use relatively simple wings, consisting of four plane elements. In the class of such wings one achieves up to 80% of the maximum gain in AP by surface deformation.

To verify the results, computations were run of the flow around flat and optimum wings (the latter consisting of 32 plane elements), using meshes with different numbers of mesh points. In each zone of the finest mesh used there were 161 and 305 mesh points in the directions of the normals to the wing plane and the plane of symmetry, respectively. In addition, 193 mesh points were positioned on the lower and upper sides of the wing bracket. The AP values obtained by extrapolating the results of the computation to a mesh of zero diameter (that is, a mesh with infinitely many mesh points) were taken as the accurate values. Then the relative increase in the AP by surface deformation was 9.6% for $M = 2$ and 9.3% for $M = 4$.

A wing of optimum form is negatively V-shaped (that is, the leading edges of the wing are deflected toward the free stream), and the wing panels are convex leeward. The wing deformation is accompanied by a redistribution of the pressure both on the surface and in the shock layer. The level curves of the pressure, in fractions of the pressure in the undisturbed flow for a flat wing and an optimum one (of 32 elements), both with $\chi = 75°$, $M = 4$, and $C_y = 0.1$, are shown in Fig. 5. The spatial position of the shock wave was not determined with any precision; it was spread out over several adjacent mesh-points. In both cases the shock wave was detached from the leading edges. In the neighbourhood of the leading edges one observes a fan of rarefaction waves. The flow, accelerating over the wing, slows down in the transverse shock wave. Note that the lift coefficient $C_y = 0.1$ in the optimum wing is obtained at a larger angle of attack than in a flat wing. This is due to the smaller size of the shock layer over a wing of optimum form. On the whole, the pressure on the optimum wing is distributed more uniformly over the wing span than on a flat wing, in which the neighbourhood of the leading edges is most heavily loaded. There is also a general increase in pressure near the plane of symmetry.

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